Probabilistic Slope Failure Risk Estimation of a Dam with Zones of Weak Foundation Material

Ryan Singh, Bob Wark

Civil Engineer, GHD

Technical Director, GHD

For existing dams built before modern theories and understanding of soil mechanics were fully developed, it was often the case that comprehensive investigations into the properties of the embankment and foundation material were not carried out. With more stringent dam safety requirements and engineering criteria, and a better understanding of soil mechanics, it is necessary to undertake embankment and foundation investigations on such dams, with the view to gain a better understanding of the embankment and foundation conditions.

This paper details the method used for a risk-based assessment of a dam's stability against slope failure for steady-state seepage conditions, based on a probabilistic assessment of differing interpretations of the material properties for the foundation. To achieve this, several separate interpretations of material strength models were developed for a foundation, using various subsets of available tri-axial data. The mean strengths of these models were used to assess the stability, and to account for the variation in strength properties of each model, the sampling distribution of the mean was used to assess the likelihood of failure.

Finally, an event-tree type risk analysis was used to calculate a value for the probability of slope failure.

A case study has been presented using this method.

Keywords: Dams, Probabilistic Assessment, Slope Stability Assessment, Event-Tree Analysis, Weak Foundation Zones

Introduction

The current understanding of soil mechanics is largely a result of the ground breaking studies and papers conducted and written by Professor Karl Terzaghi and his contemporaries in the second quarter of the 20th century. The stability analyses of dam embankments and foundations designed and constructed prior to this period were often not completed in line with modern theories and methods. These dams also often lacked comprehensive investigations of the geotechnical properties of the embankment and foundations materials.

According to modern dam safety guidelines, such as the Australian National Committee of Large Dams (ANCOLD) *Guidelines on Dam Safety Management* (2003) and *Guidelines on Risk Assessment* (2003), a risk-based approach may be taken to evaluate the safety of existing dams. As part of such an assessment, the Factors of Safety (FoS) for various failure modes, such as slope failure under steady-state seepage conditions, can be calculated and used to estimate the likelihood of failure. In the case of slope failure, the FoS and failure likelihoods are calculated according to various embankment stability calculations methods and risk relationships.

One of the key inputs into an embankment stability analysis is the selection of suitable material strength parameters which reflect the actual conditions of the embankment and foundation. In the case of existing dams, in order to do this, geotechnical investigations are necessary. Through the interpretation of in-situ and laboratory tests, such as Cone Penetration Tests and triaxial tests, a range for material parameters can be estimated. However, care must be taken in choosing material parameters based on the investigation results, as incorrect parameters can greatly affect the outcomes of the analyses.

Often a full understanding of the material properties for an existing embankment and its foundations is not possible, due to the practicalities of conducting sufficient geotechnical investigations (e.g. existing structure layout, cost, etc.) and as such, the strength properties must be determined from a limited number of sample points within the works. Where isolated zones of weak material are encountered, these can be taken into consideration when determining strength properties or, preferably, further investigation should be undertaken in targeted areas to characterise the size and location of these weak zones.

This paper presents the method developed in attempting to account for geotechnical investigations that yielded inconclusive results regarding the existence of weak zones within a foundation, using a probabilistic approach to model the likelihood of the weak zones leading to embankment failure under steady-state conditions. In addition, this led to an estimation of the likelihood of failure. This is presented based on the example of the geotechnical investigations and foundation conditions for an example dam. Additionally, the method presented is compared with other contemporary methods of estimating the probability of failure.

Note that while this method could be applied to embankment materials, due to the uncertainties of the foundation presented in the case example, this paper has focused on the analysis of foundation materials.

The example dam is a water supply dam in Western Australia, which was constructed in the 1920s. Foundation investigations for this dam which were carried out over the course of a few years in the 1980s encountered zones of weak and loose soil in the dam foundations. As part of a review of the dam in the late 1980s, the stability of the embankment was analysed, with specific interest in the locations of the weak zones encountered. The resulting failure surface through the weak zones in question is presented in Figure 1. The findings of this review led to the construction of a downstream weighting toe to aid the stability of the dam in the later 1980s.

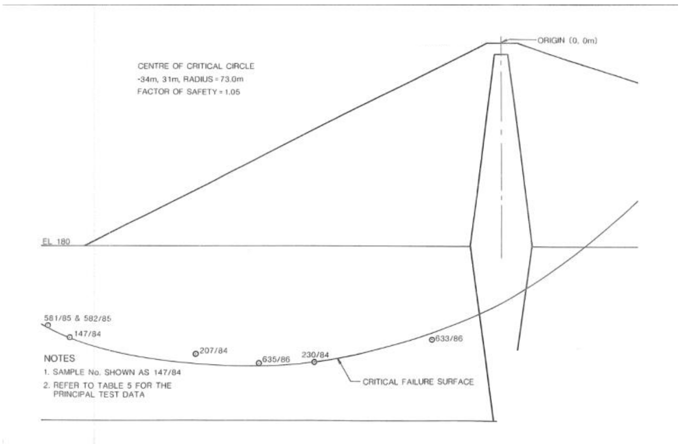

Figure 1 Stability review with failure surface through selected weak points (late 1980s)

A subsequent drilling investigation in the early 2000s, which targeted the same areas as the previous investigations, did not encounter any extensive or connected zones of weak materials in the foundation. It was concluded, however, that enough evidence from the CPT investigation existed to indicate that zones of weaker materials are present within the foundation.

Further remedial works were carried out in mid-2000s, which included the construction of a new downstream weighting toe and chimney and blanket filters. In 2016, the authors assessed the likelihood of failure taking into account the previously identified weaker materials. This paper presents the methods developed in this assessment.

Method

The method involves developing foundation strength models from various subsets of the overall foundation strength data, by determining a line-of-best-fit, for a mean strength relationship, derived for that subset, the subset being chosen that best represents the foundation conditions identified in the investigation process.

Using a normal distribution for the foundation strength model, it is possible to calculate the probability of slope failure for a dam embankment for a full range of reservoir levels. The probability of slope failure at each reservoir level is assumed to be the probability that the mean strength of the foundation model is less than the critical strength for which the FoS is unity. The total probability of slope failure for that foundation model can then be determined by summation of the failure probabilities for the range of water load partitions. This was done with the intent of directly relating the likelihood of slope failure with the chosen material strength properties, where the weaker materials were assumed to dominate the behaviour of the foundation during failure.

The use of a normal distribution for the strength models is based on the Central Limit Theorem. The theorem states that if a number of sample data sets are taken from a large population data set, and the sample size (n) of each data set is sufficiently large, the mean values of those sample data sets would have a normal probability distribution, regardless of the probability distribution of the original population data set. The Central Limit Theorem also states that if a population data set is normally distributed., we do not have to test for the convergence of the sampling distribution of the mean (i.e. there is no minimum sample size). A W-Test (Shapiro & Wilk, 1965) allows the population to be checked if it is normally distributed.

Therefore, if we have a population with a mean of $\bar{x}$ and a variance of s^2, the sampling distribution of the mean can then be approximated by:

$$\bar{x}_{sample\ mean} \approx \bar{x}_{population}$$

$$s^2_{sample\ mean} \approx \frac{s^2_{popultion}}{n}$$

where n is the sample size, i.e. the number of data points describing the sample.

The power of the sampling distribution of the mean is that in the limit, with sufficiently large n, it is independent of the actual distribution of the population.

The process is presented in Figure 2.

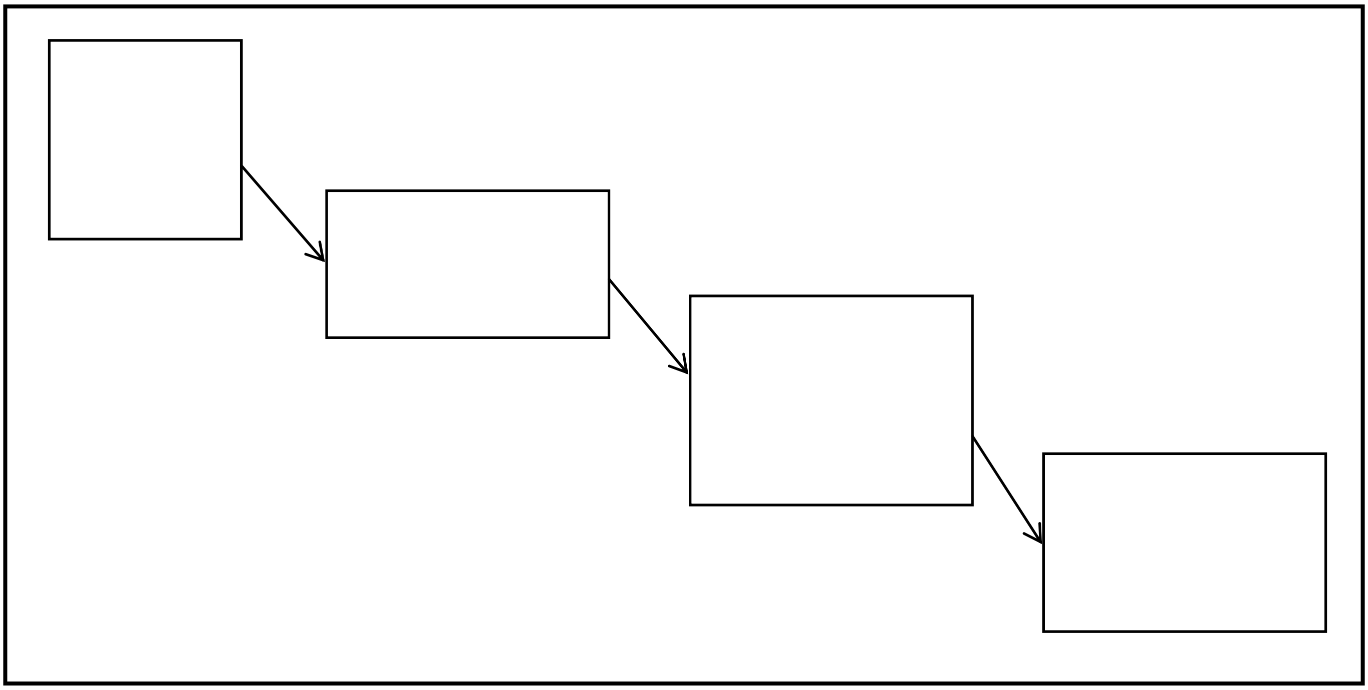

Figure 2 Method overview

Example

Foundation Model

As previously stated, the example dam has had three seperate foundation investigations which were conducted in the mid 1980s. The investigations found evidence of weak or loose dam foundation soils, concluding that there were probably isolated lenses of thick (2 m) cohesionless, loose and very wet soils. Furthermore, anecdotal evidence suggests sloppy material was encountered that was not able to be tested. In early 2000s, a review of the triaxial test results from the investigations in the 1980s found approximately 25% of the results were likely to be unreliable, due to disturbances of the samples, consolidation of the samples as they were driven and extruded, as well as difficulties with the testing apparatus. The results from the 1980s testing program produced relatively low strength results, which the early 2000s review suggested be used with caution.

A subsequent mid-2000s drilling investigation did not encounter zones of weak materials described in the previous investigations, despite targeted efforts. The findings from the mid-2000s investigation stated that the low strength results of the 1980s testing may provide a reasonable indication of the material, given the results from CPT probes that were also conducted as part of the mid-2000s investigation. The mid-2000s investigation concluded that there was enough evidence to indicate that these previous investigations did encounter the zones and that it was recommended that this should be taken into account when choosing material strength parameters.

A p’-q plot of all available data for the foundation has been presented in Figure 3, using the Cambridge Method. For reference, all data points indicated as possible unreliable in the early 2000s review have been plotted as squares, with all other accepted data as diamonds.

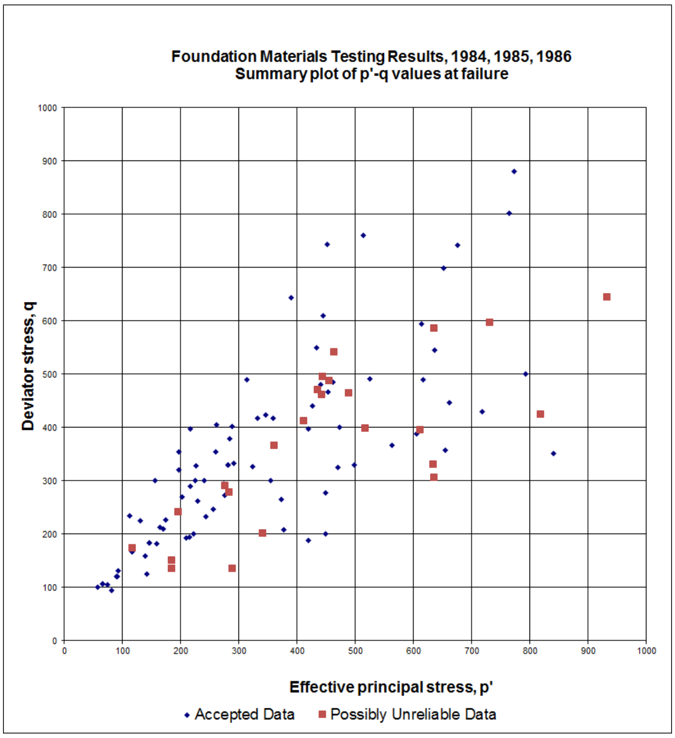

Figure 3 p'-q plot for all available foundation data

For the current review, it was decided to use all points of data, for two reasons:

- The mid-2000s geotechnical investigation stated that the evidence may not be as unreliable as previously thought when viewed with CPT results from the mid-2000s investigation
- The data points marked as unreliable appear to fit the rest of the data and did not appear to adversely bias the data one way or the other

Note that the final inclusion of this data was done as an opinion of the authors. From the data set, two lines-of-best-fit were produced; one for the whole set of data and another for the data indicating weaker results. A line-of-best-fit to represent an average strength relationship was used with the reasoning that an averaging of all weak zones within the foundation would lead to weakening of the overall foundation and would be more likely to lead to embankment failure, as opposed to one weaker zone of weak material.

The lines are presented in Figure 4.

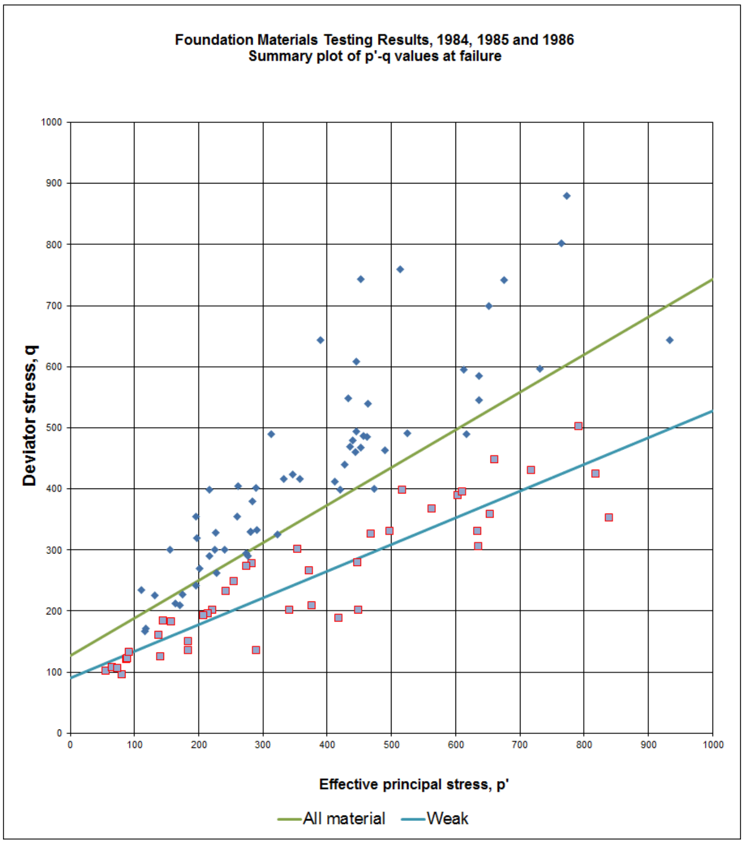

Figure 4 p'-q lines of best fit

Using equations relating the p'-q data to the tau-sigma space, the following Mohr-Coulomb equations were derived:

$$\tau' = \sigma' \tan(16.2^{\circ}) + 60$$ – All data points

$$\tau' = \sigma' \tan(11.7^{\circ}) + 43$$ – Weak data points

The models are best presented as bi-linear models, with an initial steeper friction angle before a normal stress threshold.

Table 1 Bi-linear foundation models

Model	**ϕ'_1**	**ϕ'_2**	**Normal stress threshold**
All data	30	16.2	209
Weak data	30	11.7	116

The dam embankment was analysed for stability with the above foundation strength models, using the SLOPE/W software. All other material properties for the dam embankment were assigned using recommended values from the 2003 geotechnical investigation.

The outputs from the analyses, showing the critical slope failure are presented in Figure 5 and Figure 6. The calculated FoS for the for the "All Data" foundation and "Weak" foundation were 1.83 and 1.39 respectively. For a suggested target FoS of 1.50 (Fell et. al., 2014), it can be seen that the "All Data" foundation case satisfies this; however, the "Weak" foundation case does not. As the FoS for the weak case is above 1.0, based on a deterministic assessment, the dam embankment is not expected to be at risk of failure, but would have less reserve strength capacity than required by current design standards.

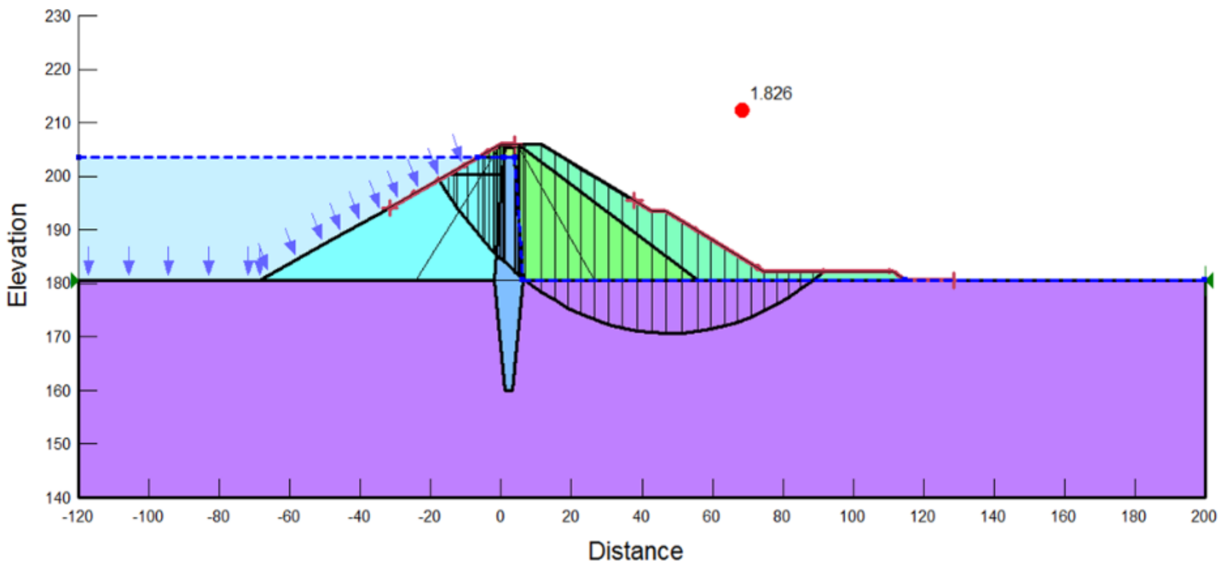

Figure 5 Critical failure slope for "All Data" Foundation

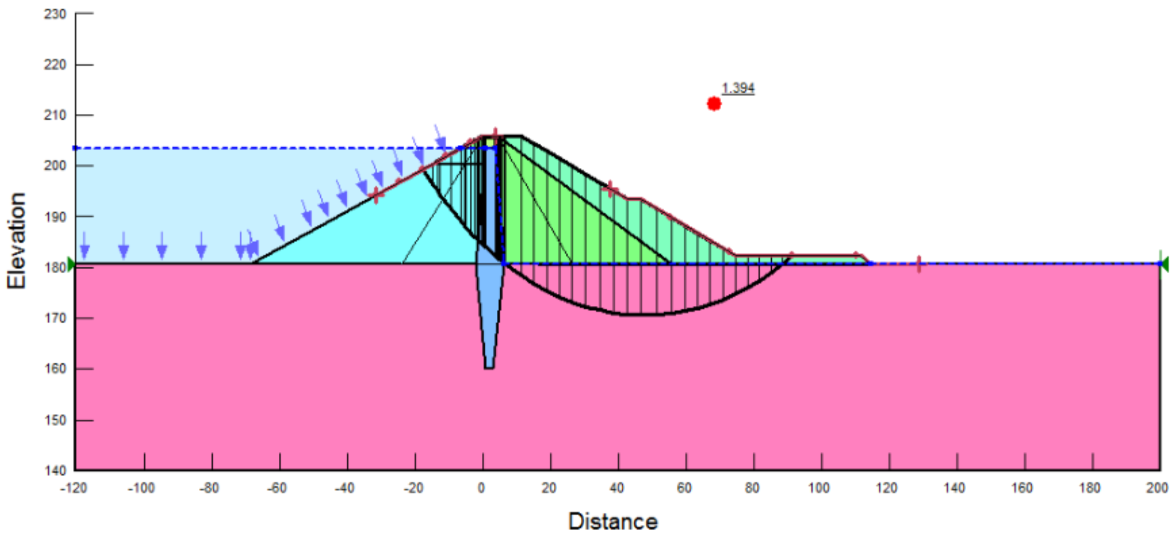

Figure 6 Critical failure slope for "Weak" Foundation

Likelihood of Failure

For the example dam, the uncertainty of the weak zones was incorporated into the likelihood of failure for steady-state conditions using the Weak Foundation model.

Referring to the method in Figure 2, Step 1 was completed with the derivation of the Weak Foundation model shown in Figure 4 and Table 1. Step 2 involves determining the properties of the sample distribution for the Weak Foundation model. Assuming the Central Limit Theorem holds, the weak subset sample mean and standard deviation can be used to estimate the sampling distribution of the mean of all the weak material, defining the mean and standard deviation of the sample means.

A check was made to see if the weak subset was best described as a normal distribution using the *analytical test for normality*, otherwise known as the W-Test (Shapiro & Wilk, 1965). For the Weak Foundation subset, there was 99% confidence that the data subset was normally distributed about the mean of the sample.

For the Weak Foundation model, ϕ'_2 can be presented as the normal distribution with a mean and variance ~ N(11.7º, 0.8^{o2}).

For Step 3 in Figure 2, the following procedure was followed:

- Analyse the dam embankment for various reservoir levels
- Determine the required ϕ'_2 for the FOS to fall below 1.0 for that reservoir level and associated steady state seepage condition, taking into account the fact that the normal stress threshold will change as ϕ'_2 changes

- Using the normal distribution properties, calculate the probability of the failure with ϕ'_2 occurring in the distribution. This can be done by using z-score standardisation to calculate the probability, or by using the probabilistic material strength parameter function in SLOPE/W. For this case study, the use of z-score standardisation was used.

Note, for failure conditions, the failure slip surface was required to pass through the phreatic surface within the embankment. This would model the loss of the embankment where the water level would be, leading to an uncontrolled release of water.

Given a failure ϕ'_2, the z-score was calculated using:

$$z = \frac{x - \mu}{\sigma}$$

where:

- z is the z-score, the number of population standard deviations away an observation is from the population mean.
- x was the ϕ'_2 for failure
- μ was the mean of the population
- σ was the standard deviation of the population

Given the Central Limit Theorem, the use of the sample mean and sample standard deviation for the "Weak" Foundation model was justified.

The probability of z occurring was then determined from standard tables and the normal probability density function. For the example dam, Table 2 presents the probability of failure ϕ'_2 calculated for the relevant reservoir partition. In addition, the annual exceedance probability of each reservoir loading being exceeded was determined based on available historical water levels for the last 35 years.

Table 2 Probabilities of ϕ'_2 for failure

WSL Loading	AEP of WSL Loading	Required ϕ'_2 FOS=1	Probability of ϕ'_2 for FOS=1	Multiplication of probabilities
Dam Crest Level	1.0×10^{-8}	6.5	4.02×10^{-11}	4.02×10^{-19}
Upper third	8.3×10^{-1}	5.3	6.20×10^{-16}	5.15×10^{-16}
Middle third	1.0	4.6	3.50×10^{-19}	3.50×10^{-19}
Lower third	1.0	4.2	3.46×10^{-21}	3.46×10^{-21}
			Total Likelihood	5.15×10^{-16}

The low probabilities for the required ϕ'_2 can be attributed to the relatively low standard deviation of the mean foundation model (0.8º).

For Step 4 (Figure 2), the probability of failure for steady-state conditions is estimated by summing the multiplication of probabilities reservoir partition and the relevant failure ϕ'_2, for example:

$$P_{\text{Middle Third}} \times P_{\text{Phi'-2 for failure}}$$

Taking into account the reservoir partition probabilities based on historical data, the final calculated likelihood of failure was **5.15×10^{-16}**. Based on the low probability, the failure mode could be considered negligible.

Comparison Without Excluded Data

In order to determine if including the previously excluded data did in fact affect the analysis, the method above was again completed for the data set, without the subset that was previously noted to be possibly unreliable.

The total set of data without the unreliable data is presented in Figure 7. Similar to above, a "Weak" subset has been selected to attempt to account for the possible weak zones of material previously encountered (square data points in the plot).

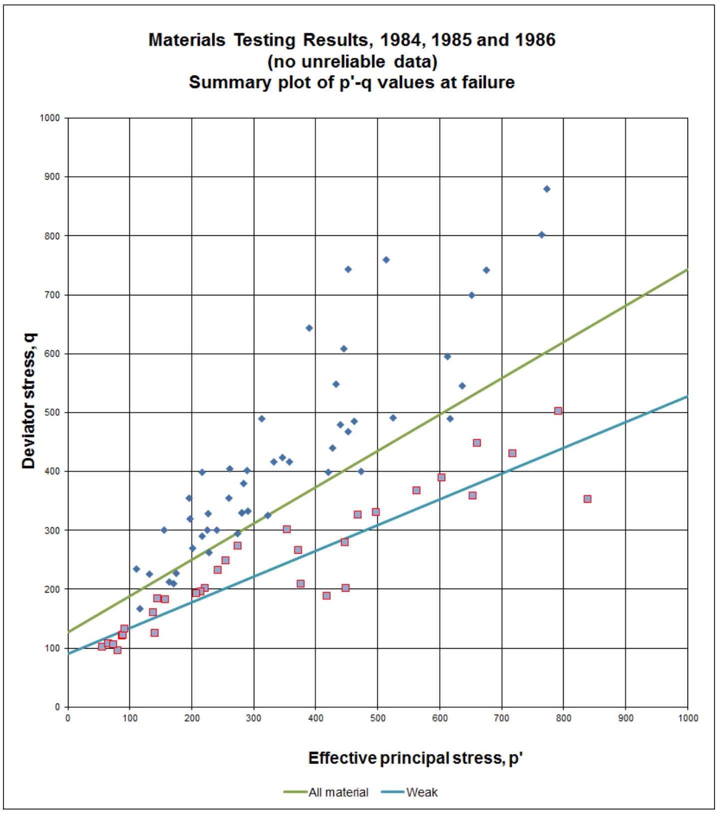

Figure 7 p'-q plot for all foundation data with possible unreliable data removed

Again, using equations relating the p'-q data to the tau-sigma space, the following Mohr-Coulomb equation for the new "Weak" foundation was calculated to be:

$$\tau' = \sigma' \tan(11.9°) + 43.9$$

This was decided to be close enough to the "Weak" foundation model with all the data points included that no meaningful difference would have been made with or without the possibly unreliable data. This justifies the inclusion of the previously rejected data, with more data points allowing a better estimate of the mean strength model.

Comparisons with Other Methods

For comparison, the likelihood of failure for steady-state conditions can be estimated using the method discussed in *Probability and Risk of Slope Failure* (Silva, Lambe and Marr, 2008). The likelihood of failure is estimated based on the FoS against slope instability and the category of the dam; how well it was built and is operated. This ranges from Category I which is described as "facilities designed, built and operated with state-of-the-practice engineering" to Category IV which is described as "facilities with little or no engineering" Figure 8 shows the relationship between the FoS and the annual probability of failure.

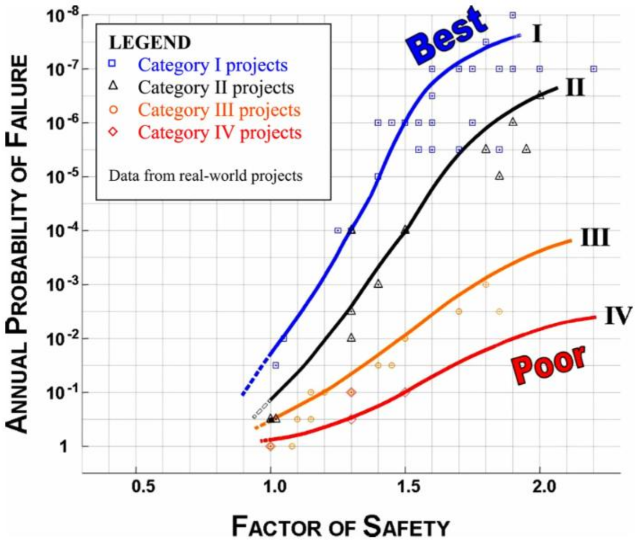

Figure 8 Factor of Safety versus annual probability of failure (Silva et al., 2008)

The example dam was considered to be a Category II structure based on the following points:

- It was a major water supply project when it was originally constructed
- Extensive geotechnical investigations for the embankment and foundation has been conducted over the last few decades
- Embankment remedial works have been carried out in the mid-2000s
- Intermediate Inspections carried out by qualified engineers
- Regular inspections carried out by trained staff

In choosing the loading condition to calculate the FoS, incorporating reservoir partition loading was considered, as was done for the method presented in this paper. However, as the method presented in Silva et al. did not use reservoir partition loading, it was decided to calculate the FoS for a Full Supply Reservoir steady-state condition. This was done for both the "All Data" Foundation Model and "Weak" Foundation Model (Figure 5 and Figure 6). The FoS calculated were 1.8 and 1.4 respectively. From Figure 8, these translate to annual probabilities of failure ranging from 3.0 x 10^{-3} and 9.0 x 10^{-5}.

The two methods have given widely varying numerical answers, based on notably different methods and assumptions. The method presented in this paper attempts a probabilistic approach based on site specific material strengths, whereas the method presented in Silva et al. has used a plethora of case studies with stability analyses with experienced engineering judgment to assign subjective probabilities.

The argument for the method presented in this paper is that is takes into account site specific conditions in the determination of the probability of failure using a probabilistic assessment. The method in Silva et al. allows an estimation of the probability of failure, based on previous case studies that are similar in characteristics to the example in question, possibly leading to an overestimation in the probability of failure.

Where some geotechnical information is available for the embankment and foundation, it may be the case that method presented in this paper calculates a more rigorous estimate for the probability of failure.

Conclusion

A probabilistic method for determining the risk of slope failure for an earthfill dam embankment where known weak zones within the foundation exist was applied. The method attempted to directly relate the conditions at the site directly to the probability of failure by modelling the mean strength of the foundation as a normal distribution. An example was shown for an example dam, where the probability of failure for steady-state conditions was estimated to be 1.15 x 10^{-16} for a lower bound "Weak" foundation model.

The likelihood of failure estimated from all the information known about the geotechnical properties of the example dam is very low. This method is dependent on what is known about the foundation conditions existing at the site. The estimate of the failure probability is dictated by the unknowns and uncertainties of the actual foundation conditions, such as how extensive the weak zones are, how they are located spatially within the foundation and whether these uncertainties may have to be described by other mechanisms. However, while the FoS found is below the recommended value, the probability of failure is acceptably low.

Acknowledgments

The authors wish to acknowledge the Dams and Dam Safety Section, Water Corporation, Western Australia, for allowing the use of their data in this paper.

References

Australian National Committee on Large Dams. 2003. Guidelines on Dam Safety Management

Australian National Committee on Large Dams. 2012. Guidelines on Risk Assessment.

Fell R., MacGregor, P., Stapledon, D., Bell, G., Foster, M. 2014. Geotechnical Engineering of Dams. *CRC Press*.

Engineering Geology Report Dam Safety Review Investigations 1986 Geological Report

Engineering Geology Report Safety Review Investigations 1984 and 1985 Geological Report.

Shapiro, S. S.; Wilk, M. B. 1965. An Analysis of Variance Test for Normality. *Biometrika 52 (3-4): 591-611*

Silva, F.; Lambe, T. W.; Marr, W. A. 2008. Probability and Risk of Slope Failure. *Journal of Geotechnical and Geoenvironmental Engineering 2..8.134:1691-1699*

1987 Dam Safety Review.

2003 Dam Safety Review Geotechnical Assessment Report.